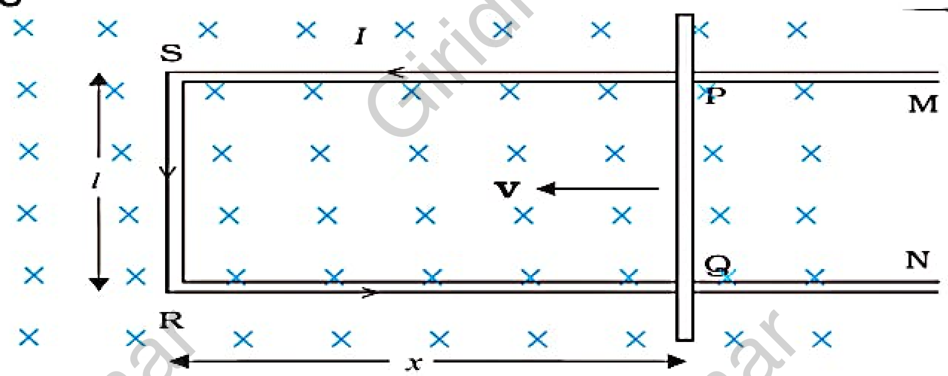
Energy Consideration: A Quantitative Study



As we know, Motional E.M.F induced in the rod will be

$$\mathcal{E} = Bl V$$

Now, the resistance of rod is 'r'

$\therefore$ current in the rod will be

$$i = \frac{E.M.F}{resistance}$$

$$\boxed{i = \frac{BlV}{r}}$$

According to Lenz's Law, the direction of current will be PQRS.

here, Rod 'PQ' will experience a magnetic force towards right because we are trying to move the rod left.

This magnetic force will be

$$\vec{F_B} = i(\vec{l} \times \vec{B})$$

$$\therefore F_B = i l B \sin 90^\circ$$

$$F_B = \frac{Blv}{r} \times l B$$

$$F_B = \frac{B^2 l^2 v}{r}$$

due to this magnetic force, applied velocity will try to decrease but in order to maintain the speed constant we must apply the power

$$\text{power} = \text{force} \times \text{velocity} \quad \left\{ \begin{array}{l} \text{we know} \\ P = F \times V \end{array} \right\}$$

$$= \frac{B^2 l^2 v}{r} \times v$$

$$\boxed{P = \frac{B^2 l^2 v^2}{r}}$$

This applied power will now dissipate at the resistance of rod and the work done by applied force will be converted in the form of heat

$$\text{power dissipated at resistance of rod} = i^2 r$$

$$= \left(\frac{Blv}{r}\right)^2 \times r$$

$$= \frac{B^2 l^2 v^2}{r^{\cancel{2}}} \times \cancel{r}$$

$$\boxed{P = \frac{B^2 l^2 v^2}{r}}$$

$\because$ Applied Power = Electrical power

$\therefore$ We can say mechanical energy which was needed to move the arm 'PQ' is converted into electrical energy and then converted into heat energy.

$\longrightarrow$ This is Energy Consideration that energy remains conserved.

Note :-

$$\because \quad \mathcal{E} = \frac{\Delta \phi_B}{\Delta t} \quad \text{—(i)} \quad \left(\text{from Faraday's law}\right)$$

$$\& \quad \mathcal{E} = ir = \frac{\Delta Q}{\Delta t} \times r \qquad \left(\because i = \frac{\mathcal{E}}{r} \;\&\; i = \frac{\Delta Q}{\Delta t}\right)$$

current

$$\therefore \mathcal{E} = \frac{\Delta Q}{\underbrace{\Delta t}_{\text{time taken}}} r \quad \text{—(ii)}$$

from (i) & (ii)

$$\frac{\Delta Q}{\cancel{\Delta t}} \times r = \frac{\Delta \phi_B}{\cancel{\Delta t}}$$

Change in Charge

$$\boxed{\Delta Q = \frac{\Delta \phi_B}{r}}$$

Change in flux

resistance